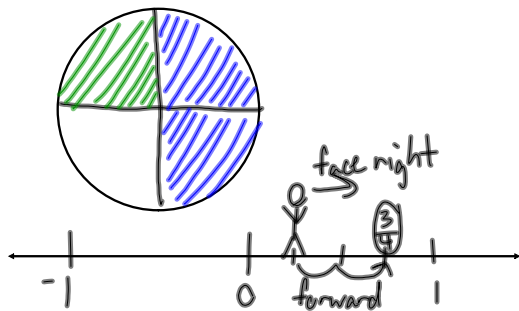


6.2 Completed Notes

6.2: Addition and Subtraction of Rational Numbers

Definition: If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

Example: Draw a figure and a number line to represent $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$.



Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof:

$$\frac{a}{b} = \frac{a \cdot d}{b \cdot d} = \frac{ad}{bd}, \quad \frac{c}{d} = \frac{c \cdot b}{d \cdot b} = \frac{bc}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Definition: A mixed number is a number of the form $a\frac{b}{c}$, where a is an integer and $\frac{b}{c}$ is a proper fraction. The notation means $a\frac{b}{c} = a + \frac{b}{c}$.

A mixed number is a rational number, so we should be able to write it as $\frac{a}{1}$.

Example: Write the following numbers in $\frac{a}{b}$ form.

$$(a) 2\frac{1}{4} = 2 + \frac{1}{4} = \frac{2}{1} + \frac{1}{4} = \frac{2 \cdot 4}{1 \cdot 4} + \frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$$

$$\left(2\frac{1}{4}\right) = \frac{9}{4}$$

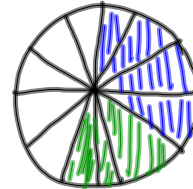
$$(b) -3\frac{2}{5} = -\left(3 + \frac{2}{5}\right) = -3 - \frac{2}{5} = \frac{-15}{5} - \frac{2}{5} = \frac{-17}{5}$$

$$-\left(3\frac{2}{5}\right) = \frac{-17}{5}$$

Example: Find a method of evaluating $\frac{1}{3} + \frac{1}{4}$.

$$\frac{1}{3} = \frac{1 \cdot 4}{3 \cdot 4} = \frac{4}{12}, \quad \frac{1}{4} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}$$

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{4+3}{12} = \frac{7}{12}$$



Example: Calculate the following sums. Simplify your answers.

$$(a) \frac{3}{10} + \frac{4}{15} = \frac{3 \cdot 3}{10 \cdot 3} + \frac{4 \cdot 2}{15 \cdot 2} = \frac{9}{30} + \frac{8}{30} = \frac{17}{30}$$

$$\frac{3}{10} + \frac{4}{15} = \frac{45+40}{150} = \frac{85}{150} = \frac{17}{30}$$

$$(b) \frac{3}{9} + \frac{2}{6} = \frac{3 \cdot 2}{9 \cdot 2} + \frac{2 \cdot 3}{6 \cdot 3} = \frac{6}{18} + \frac{6}{18} = \frac{12}{18} = \frac{2}{3}$$

$$\frac{3}{9} + \frac{2}{6} = \frac{18+18}{54} = \frac{36}{54} = \frac{2}{3}$$

$$(\text{reduce first}) \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Example: Change the following fractions to mixed numbers.

$$(a) \frac{22}{7} = 3\frac{1}{7}$$

$$\begin{array}{r} 3 \\ 7 \overline{) 22} \\ \underline{-21} \\ 1 \end{array}$$

wholes
denominator
numerator

$$(b) \frac{64}{19} = 3\frac{7}{19}$$

$$\begin{array}{r} 3 \\ 19 \overline{) 64} \\ \underline{-57} \\ 7 \end{array}$$

6.2 Completed Notes

Example: Calculate the following sums. Leave your answers as mixed numbers.

$$(a) 2\frac{3}{7} + 1\frac{11}{14} = (2 + \frac{3}{7}) + (1 + \frac{11}{14})$$

$$= (2+1) + (\frac{3}{7} + \frac{11}{14}) = (2+1) + (\frac{6}{14} + \frac{11}{14})$$

$$= 3 + \frac{17}{14} = 3 + 1\frac{3}{14} = 4\frac{3}{14}$$

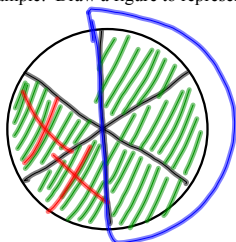
$$(b) 2\frac{1}{2} + 3\frac{2}{3}$$

$$\begin{array}{r} 2\frac{1}{2} \rightarrow 2\frac{3}{6} \\ + 3\frac{2}{3} \rightarrow + 3\frac{4}{6} \\ \hline 5 + \frac{7}{6} = \boxed{6\frac{1}{6}} \end{array}$$

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is the unique rational number $\frac{e}{f}$ such that $\frac{a}{b} = \frac{c}{d} + \frac{e}{f}$.

Note: An easy formula that we can get from this is $\frac{a}{b} - \frac{c}{d} = \frac{a-d}{c}$.

Example: Draw a figure to represent $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.



Example: Calculate the following. Simplify your answer.

$$(a) \frac{5}{6} - \frac{1}{2} = \frac{12-5}{20} = \boxed{\frac{7}{20}}$$

$$(b) \frac{13}{16} - \frac{7}{30} = \frac{13 \cdot 15}{16 \cdot 15} - \frac{7 \cdot 8}{30 \cdot 8} = \frac{195}{240} - \frac{56}{240} = \frac{139}{240}$$

LCM is 240

Which of our number properties are represented in the rational numbers over addition?

Closure: $\frac{a}{b} + \frac{c}{d} \in \mathbb{Q}$

Commutative: $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

Associative: $(\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} = \frac{a}{b} + (\frac{c}{d} + \frac{e}{f})$

Identity: $\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$

Inverse: $\frac{a}{b} + \frac{-a}{b} = \frac{-a}{b} + \frac{a}{b} = 0$

Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof: $\frac{a}{b} = \frac{a \cdot d}{b \cdot d} = \frac{ad}{bd}, \frac{c}{d} = \frac{c \cdot b}{d \cdot b} = \frac{bc}{bd}$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$$

Example: Calculate the following. Simplify your answer.

$$(a) 2\frac{1}{6} - 1\frac{9}{20}$$

$$\begin{array}{r} 2\frac{1}{6} \rightarrow 2\frac{10}{60} \\ - 1\frac{9}{20} \rightarrow - 1\frac{27}{60} \\ \hline 1\frac{43}{60} \end{array}$$

$$(b) 3\frac{1}{2} - 1\frac{5}{8}$$

$$\begin{array}{r} 3\frac{1}{2} \rightarrow 3\frac{4}{8} \\ - 1\frac{5}{8} \rightarrow - 1\frac{5}{8} \\ \hline 2\frac{7}{8} \end{array}$$

6.2 Completed Notes

$$4\frac{2}{7} - 1\frac{5}{14}$$

$$\begin{array}{r} 3\cancel{4}\frac{\cancel{4}^{18}}{14} \\ -1\frac{5}{14} \\ \hline 2\frac{13}{14} \end{array}$$